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New divergences of tachyon in the two-dimensional charged black holes

H.W. Lee and Y. S. Myung

Department of Physics, Inje University, Kimhae 621-749, Korea

Jin Young Kim

Division of Basic Science, Dongseo University, Pusan 616-010, Korea

D. K. Park

Department of Physics, Kyungnam University, Masan 631-701, Korea

Abstract

We quantize the tachyon field in the two-dimensional (2D), $\epsilon < 2$ charged black holes where ϵ is the dilaton coupling parameter for the Maxwell term. Especially the expectation value of the stress-energy tensor $\langle T_{ab} \rangle$, observed by a freely falling observer, is computed. This shows that new divergences such as $\ln f$ and $\frac{1}{f}$ arises near the horizon ($f \to 0$), compared with conformal matter case.

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Lower dimensional theories of gravity provide the simplified contexts in which to study black hole physics [1]. The non-triviality of these models arises from the non-minimal coupling of the dilaton to the scalar curvature. A dilaton potential of the type produced by the string loop corrections may induce multiple horizons [2]. For example, 2D charged black hole from heterotic string theories has shown this feature. This has many analogies with the Reissner-Nordström black hole in 4D general relativity.

It is important to investigate the classical stability of the black holes, which is essential to establish their physical existence [3]. It has been shown that the 2D dilaton black hole is stable [4], while the extremal black holes are shown to be classically untable [5]. Furthermore the quantum stability is also important [6], since the back reaction effects due to the non-zero stress-energy of the quantum field will change the background geometry of space-time near the event horizon. Even if it is classically stable, the instability may be caused by the divergence of the renormalized expectation value of the the stress-energy tensor($\langle T_{\mu\nu}\rangle$) associated with the quantized matter field. If one finds a divergence of the stress-energy, the solution to the quantum theory does not exist near the classical solution and thus the quantum effects alter drastically the classical spacetime geometry [7]. As a result, the black hole is unstable quantum mechanically if there exists a divergence of the stress-energy. A conformally invariant scalar field (f_i) is usually used to study the quantum aspects of black hole [8]. If one takes a conformally invariant matter to study the classical aspects of the black hole, one finds the free field equation for the perturbation : $\nabla^2 f_i = 0 \to (d^2/dr^{*2} + \omega^2) f_i = 0$. This implies that one cannot find the potential, which is crucial for obtaining information about the 2D black hole. Although f_i is a simple matter field for the quantum study of the black hole, it does not include all information for the 2D black holes.

Here we introduce a tachyon as a test field. This provides us the potential that illustrates many qualitative results about the 2D charged black holes. Further new quantum results are expected, because the tachyon is coupled nontrivially to dilation. In this paper, we consider the two-dimensional dilaton gravity coupled to Maxwell and tachyon fields. The relevant coupling (parametrized by ϵ) between the dilaton and Maxwell field is included to obtain

the general 2D, $\epsilon < 2$ charged black holes. This may be considered as a two-dimensional counterpart of the 4D dilaton gravity with the parameter a [9].

Trivedi [10] showed that the stress-energy tensor of a conformal matter diverges at the horizon in the 2D, $\epsilon = 0$ extremal black hole. This divergence can be better understood by regarding the extremal black hole as the limit of the non-extremal one. The non-extremal black hole has both the outer (event) and inner (Cauchy) horizons, and the two horizons come together in the extremal limit. In this case, it is found that if one adjusts the quantum state of the scalar field so that the stress-energy tensor is finite at the outer horizon, it always diverges at the inner horizon.

We start with two-dimensional dilaton (Φ) gravity conformally coupled to Maxwell ($F_{\mu\nu}$) and tachyon (T) fields [2,5,12]

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-G} e^{-2\Phi} \{ R + 4(\nabla\Phi)^2 + 4\lambda^2 - \frac{1}{2} e^{\epsilon\Phi} F^2 - (\nabla T)^2 - V(T) \} \tag{1}$$

with the tachyon potential $V(T) = -m_0^2 T^2$. Our sign conventions and notation follow Misner, Thorne, and Wheeler [11]. The above action with $\epsilon = 0$ can be realized from the heterotic string. Then the equations of motion become

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - \nabla_\mu T \nabla_\nu T - \frac{4-\epsilon}{4} e^{\epsilon\Phi} F_{\mu\rho} F_\nu{}^\rho = 0, \tag{2}$$

$$\nabla^2 \Phi - 2(\nabla\Phi)^2 + \frac{1}{4} e^{\epsilon\Phi} F^2 + \frac{m_0^2}{2} T^2 + 2\lambda^2 = 0, \tag{3}$$

$$\nabla_\mu F^{\mu\nu} - (2-\epsilon)(\nabla_\mu \Phi) F^{\mu\nu} = 0, \tag{4}$$

$$\nabla^2 T - 2\nabla\Phi \cdot \nabla T + m_0^2 T = 0. \tag{5}$$

The general solution to (2)-(5) with tachyon condensation is given by

$$\bar{\Phi} = -\lambda r, \quad \bar{F}_{tr} = Q e^{-(2-\epsilon)\lambda r}, \quad \bar{T} = 0, \quad \bar{G}_{\mu\nu} = \begin{pmatrix} -f & 0 \\ 0 & f^{-1} \end{pmatrix} \tag{6}$$

with

$$f = 1 - \frac{M}{\lambda} e^{-2\lambda r} + \frac{Q^2}{2\lambda^2(2-\epsilon)} e^{-(4-\epsilon)\lambda r}, \tag{7}$$

where M and Q are the mass and electric charge of the black hole, respectively. Note that from the requirement of $\bar{F}(r \to \infty) \to 0$ and $f(r \to \infty) \to 1$, we have the important constraint : $\epsilon < 2$. Hereafter we take $M = \lambda = \sqrt{2}$ for convenience. In the non-extremal black hole, from $f = 0$ we obtain two roots $(r_\pm)$ where $r_+(r_-)$ correspond to the event (Cauchy) horizon. The extremal black hole may provide a toy model to investigate the late stages of Hawking evaporation [13]. This is recovered from the non-extremal black hole in the extremal limit $(Q \to M : r_- \to r_+ \equiv r_o)$. For $\epsilon < 2$, the shape of f is always concave. The multiple root$(r = r_o)$ is thus obtained when $f(r_o) = 0$ and $f'(r_o) = 0$, in this case the electric charge of the black hole is $Q_e^2 = 8(\frac{2-\epsilon}{4-\epsilon})^{(4-\epsilon)/2}$. Here the prime ($\prime$) denotes the derivative with respect to r. The extremal horizon is located at

$$r_o(\epsilon) = -\frac{1}{2\sqrt{2}} \log(\frac{4-\epsilon}{2-\epsilon}). \tag{8}$$

The explicit form of the extremal f is

$$f_e(r,\epsilon) = 1 - e^{-2\sqrt{2}r} + \frac{2}{(2-\epsilon)}(\frac{2-\epsilon}{4-\epsilon})^{(4-\epsilon)/2} e^{-(4-\epsilon)\sqrt{2}r}. \tag{9}$$

Now let us briefly review the classical aspects of our model. We introduce small perturbation fields around the background solution as

$$F_{tr} = \bar{F}_{tr} + \mathcal{F}_{tr} = \bar{F}_{tr}[1 - \frac{\mathcal{F}(r,t)}{Q}], \tag{10}$$

$$\Phi = \bar{\Phi} + \phi(r,t), \tag{11}$$

$$G_{\mu\nu} = \bar{G}_{\mu\nu} + h_{\mu\nu} = \bar{G}_{\mu\nu}[1 - h(r,t)], \tag{12}$$

$$T = \exp(\bar{\Phi})[0 + \tilde{T}(r,t)]. \tag{13}$$

One has to linearize (2)-(5) in order to obtain the equations governing the perturbations. However, the classical stability should be based on the physical degrees of freedom. It is thus important to check whether the graviton (h), dilaton (ϕ), Maxwell mode ($\mathcal{F}$) and tachyon (t) are physically propagating modes in the 2D charged black hole background. According to the counting of the degrees of freedom, the gravitational field ($h_{\mu\nu}$) in D-dimensions has $(1/2)D(D-3)$. For the 4D Schwarzschild black hole, we obtain two degrees

of freedom. These correspond to the Regge-Wheeler mode for odd-parity perturbation and Zerilli mode for even-parity perturbation [3]. We have -1 for $D = 2$. This means that in two dimensions the contribution of the graviton is equal and opposite to that of a spinless particle (dilaton). The graviton-dilaton modes $(h + \phi, h - \phi)$ are gauge degrees of freedom and thus are nonpropagating modes. In addition, the Maxwell field has $D - 2$ physical degrees of freedom. The Maxwell field has no physical degrees of freedom for $D = 2$. Since all these fields are nonpropagating modes, equations (2)-(4) are not essential for our study.

On the other hand, the tachyon is a physically propagating mode. This is described by (5) and (13). Its linearized equation can be expressed in terms of $\tilde{T}$ as

$$\nabla^2 \tilde{T} - [(\nabla \bar{\Phi})^2 - \nabla^2 \bar{\Phi} - m_0^2] \tilde{T} = 0. \tag{14}$$

From this one finds

$$f^2 \tilde{T}'' + f f' \tilde{T}' - f[\lambda f' + \lambda^2 f - m_0^2] \tilde{T} - \partial_t^2 \tilde{T} = 0. \tag{15}$$

To study the classical stability, we should transform the above equation into one-dimensional Schrödinger equation by introducing the tortoise coordinate (r^*)

$$r \rightarrow r^* \equiv g(r). \tag{16}$$

Requiring that the coefficient of the linear derivative vanish, one finds the relation

$$g' = \frac{1}{f}. \tag{17}$$

Assuming $\tilde{T}(r^*, t) \sim \tilde{T}(r^*) e^{i\omega t}$, one can cast (15) into the Schrödinger equation

$$\{ \frac{d^2}{dr^{*2}} + \omega^2 - V(r) \} \tilde{T} = 0, \tag{18}$$

where the effective potential $V(r)$ is given by

$$V(r) = f\{(\nabla \bar{\Phi})^2 - \nabla^2 \bar{\Phi} - m_0^2\} = f\{\lambda^2 f + \lambda f' - m_0^2\}. \tag{19}$$

Note that one finds $V(r) = 0$ for a conformally invariant matter (f_i). For $m_0^2 = \lambda^2 = 2$, it is found that all 2D extremal black holes are classically unstable [5]. Furthermore the outer horizon of 2D non-extremal black hole is stable, while the inner horizon is unstable.

Now we are in a position to discuss the quantum stability. First of all we have to evaluate the one-loop effective action of the tachyon. From (1) and (13), the relevant action for the tachyon is rewritten in terms of $\tilde{T}$ as

$$S_{\tilde{T}} = -\frac{1}{2\pi} \int d^2x \sqrt{-G} \{ (\nabla \tilde{T})^2 + [(\nabla \Phi)^2 - \nabla^2 \bar{\Phi} - m_0^2] \tilde{T}^2 \}. \tag{20}$$

The linearized equation (14) is also derived from the above action. The coupling of the tachyon to the dilaton is separated as

$$(\nabla \bar{\Phi})^2 - \nabla^2 \bar{\Phi} - m_0^2 = \mathcal{Q} + m^2, \tag{21}$$

where

$$\mathcal{Q} = \frac{V(r)}{f} = (\nabla \bar{\Phi})^2 - \nabla^2 \bar{\Phi} - m_0^2; \qquad m^2 = \lambda^2 - m_0^2. \tag{22}$$

We are interested in the massless tachyon, which corresponds to $\lambda^2 = m_0^2$. Quantizing the tachyon in the background of (6) and (7) leads to the one-loop effective action. Keeping terms quadratic in the classical fields R and $\mathcal{Q}$, the relevant nonlocal part is given by [12]

$$\Gamma_{nloc} = -\frac{1}{8\pi} \int d^2x \sqrt{-G} \{ \frac{1}{12} R \frac{1}{\nabla^2} R - \mathcal{Q} \frac{1}{\nabla^2} R + \mathcal{Q} \beta^{(1)} \mathcal{Q} \}, \tag{23}$$

where

$$\beta^{(1)} = \frac{1}{\nabla^2} \lim_{\gamma \to 1} \ln \frac{1 + \sqrt{\gamma}}{1 - \sqrt{\gamma}}, \tag{24}$$

with

$$\gamma = \frac{1}{1 - \frac{4m^2}{\nabla^2}}. \tag{25}$$

The last term is the nonlocal infrared divergence, encounted in the massless limit ($m^2 \to 0$) [14]. Using the ζ-function regularization, the logarithmic divergences do not appear but one finds the renormalization papameter μ of the nonlocal infrared divergences. Here we will not consider the last term, since this depends on the renormalization parameter μ. For $\mathcal{Q} = m = 0$, we get only the first term, which is the well-known result for a conformal

matter [8]. For $\mathcal{Q} \neq 0$ and $m = 0$, the first two-terms contribute the conformal anomaly. This calculation is easily done in the conformal gauge, rather than the Schwartzschild gauge in (6). Using the tortoise coordinate r^* in (16), the line element is given by $ds^2 = f(-dt^2 + dr^{*2})$. Comparing this with the general conformal form ($ds^2 = e^{2\rho}(-dt^2 + dr^{*2})$), one finds that $\rho = \frac{1}{2}\ln f$. Using this relation, the first two-terms become local and the trace anomaly is easily computed as

$$\langle T^\alpha_\alpha \rangle = -\frac{f''}{24\pi} - \frac{\lambda^2}{4\pi}(f + 2f\ln f - \ln f - 1), \tag{26}$$

where the last expression is the new contribution, due to the tachyon coupling to dilaton. The conservation of stress-energy in the Schwarzschild gauge in (6) leads to

$$\langle T^r_t \rangle = C_1; \quad \langle T^r_r \rangle = \frac{C_2}{f} + \frac{1}{2f}\int^r_{r_+} dr f' \langle T^\alpha_\alpha \rangle, \tag{27}$$

where two integration constants C_1 and C_2 can be determind by considering all information about the quantum state of the field [15]. For simplicity, we choose $C_1 = C_2 = 0$. Substituting (26) into (27), then one can perform the integration to find

$$\langle T^r_r \rangle = -\frac{f'^2 - f'^2(r_+)}{96\pi f} - \frac{\lambda^2}{8\pi}(f-1)\ln f. \tag{28}$$

Now we turn to the issue of the regularity of stress-energy on the horizon. Since the line element($ds^2 = -f dt^2 + \frac{1}{f}dr^2$) is singular on the horizon, we introduce a freely falling observer. We want to claculate the stress-energy components in the orthonormal frame attached to a freely falling observer (FFO). The basis vectors of the frame are chosen to be the two-velocity ($e^\alpha_0 = u^\alpha$) and a unit length spacelike vector ($e^\alpha_1 = n^\alpha$) orthogonal to u^α. The components of the tress-energy tensor $\langle T_{ab} \rangle$ in the orthonormal frame are given in terms of the coordinate components as

$$\rho = u^\alpha u^\beta \langle T_{\alpha\beta} \rangle = E^2 F(r) - \langle T^r_r \rangle; \quad p = n^\alpha n^\beta \langle T_{\alpha\beta} \rangle = E^2 F(r) - \langle T^t_t \rangle, \tag{29}$$

where E is the energy per unit mass along the time-like geodesic of FFO and the quantity $F(r)$ is given by

$$F(r) = \frac{\langle T^r_r \rangle - \langle T^t_t \rangle}{f}. \tag{30}$$

The stress-energy tensor is regular on the horizon only if $\langle T^r_r \rangle$, $\langle T^t_t \rangle$ and $F(r)$ are each separately finite at outer horizon $r = r_+$. Since the strongest possible divergence comes from $F(r)$, we analyze the divergence structure of this term. This is computed near the horizon as

$$\lim_{r \to r_+} F(r) = \frac{1}{48\pi} \lim_{r \to r_+} \frac{f'''}{f'} + \frac{\lambda^2}{4\pi} \lim_{r \to r_+} (\ln f - \frac{1}{f} + 1). \tag{31}$$

The first term was discovered by Trivedi for the extremal black holes [10]. From (8) and (9), one finds $f(r_o) = f'(r_o) = 0$ for the extremal black holes and thus one has a weak divergence. In additon, here we find the new divergences from the last term. These are the divergences of the form $\ln f$ and $\frac{1}{f}$ as $f \to 0$ (near the horizon). These divergences remain for the non-extremal black holes too, although the form is softened. Since $f = 0$ has a multiple root for extremal black holes, the divergence is stronger than that of non-extremal black holes.

In conclusion, we find the new divergences which induce the quantum instability of both the extremal and non-extremal black holes.

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